Local4Global

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Overview: ETHZ Contributions

D3.3 (M1-M12): *Optimality, Certification and Stability in TSoS*

Deliverable includes:

- Characterization of the optimization problems
- Concepts from ADP, as a tool for model-free control
- Approximation for decision making under uncertainty[1]
- Concepts from distributed optimization, emphasis ADMM

[1] Binary Decision Rules for Multistage Adaptive Mixed-Integer Optimization, Georghiou, A. and Bertsimas, D. (under review MPA)

Overview: ETHZ Contributions

D4.1.1 (M10-M18): *TSoS Distributed Optimizer (Part I)*

Deliverable includes:

- Formulate and solve a building control problem (using D3.3)[2]
- Novel technique for chance-constrained problems[3]

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[2] A Stochastic Optimization Approach to Cooperative Building Energy Management via an Energy Hub, Darivianakis, G., Georghiou, A., Smith, R. S. and Lygeros, J. (under review IEEE CDC)

[3] Tractable Approximation of Chance-Constrained Systems via Piecewise Affine Policies, Zhang, X., Georghiou, A., and Lygeros, J.(under review IEEE CDC)

Overview: ETHZ Contributions

D4.1.2 (M10-M24): *TSoS Distributed Optimizer (Part II)*

Deliverable includes:

- Use ADP to design policies with distributed structure[4]
- Decentralized control using decision rules in DP framework (using D3.3)[5]

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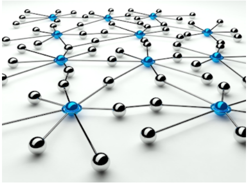
[4] Building Control using Distributed Approximate Dynamic Programming, (working paper)

[5] The Decision Rule Approach to Dynamic Programming (working paper)

Distributed Optimization

Necessary tools to achieve goal:

- High quality control design for local problems
- Solution method using distributed optimization



Addressing problems affected by uncertainty

Optimization Model

Objectives

- Minimize operating costs
- Occupant satisfaction constraints
- Design closed-loop policies *"Rule Based Control"*

$$
\begin{aligned}
\text{minimize} \quad & \mathbb{E}\left(\sum_{t=1}^{T} \ell_t(\boldsymbol{x}_t, \boldsymbol{u}_t)\right) \\
\text{subject to} \quad & \left.\begin{aligned} &\boldsymbol{u}_t = \boldsymbol{u}_t(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_t) \\ &\boldsymbol{x}_t \in \mathcal{X}_t,\ \boldsymbol{u}_t \in \mathcal{U}_t, \\ &\boldsymbol{x}_{t+1} = f_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t), \end{aligned}\right\} \forall \underbrace{\boldsymbol{\xi} \in \Xi}_{\text{uncertainty}},\ t \in \mathcal{T}.
\end{aligned}
$$

Building Dynamics

Building Resistance-Capacitance Model[6]

Automatic generation

- Building data into bilinear state-space model
- Costs and constraints

$$x_{t+1} = Ax_t + Bu_t + C\xi_t + \sum_{i=1}^{n}(D_i\xi_t + E_ix_t)u_{i,t}$$

Model Validation

- Against EnergyPlus (difference $< 0.5\ ^oC$)
- Against 5 story building (difference $< 0.5\ ^oC$)

6 BRCM Toolbox, ETH Zürich

Optimization Model

For linear dynamical systems

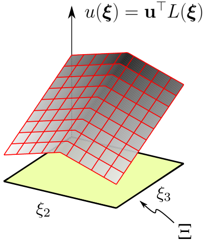
$$\overbrace{\boldsymbol{u}_t(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_t)}^{\text{State feedback}} \iff \overbrace{\boldsymbol{u}_t(\boldsymbol{\xi}_1,\ldots,\boldsymbol{\xi}_t)}^{\text{Disturbance feedback}}$$

$$\begin{array}{ll} \text{minimize} & \mathbb{E}\left(\sum_{t=1}^{T} \ell_t(\boldsymbol{x}_t,\boldsymbol{u}_t)\right) \\ \text{subject to} & \left.\begin{array}{l} \boldsymbol{u}_t = \boldsymbol{u}_t(\boldsymbol{\xi}_1,\ldots,\boldsymbol{\xi}_t), \\ \boldsymbol{x}_t \in \mathcal{X}_t,\ \boldsymbol{u}_t \in \mathcal{U}_t, \\ \boldsymbol{x}_{t+1} = f_t(\boldsymbol{x}_t,\boldsymbol{u}_t,\boldsymbol{\xi}_t), \end{array}\right\} \forall \underbrace{\boldsymbol{\xi} \in \Xi}_{\text{uncertainty}},\ t \in \mathcal{T}. \end{array}$$

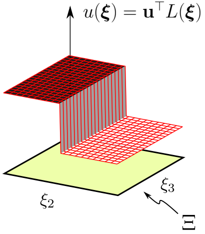
Decision Rule Approximation[7]

Affine Feedback Control Design

$$u_t(\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_t) = \mathrm{u}_{t,0} + \sum_{s=1}^{t} \mathbf{u}_{t,s}^\top L_s(\boldsymbol{\xi}_s) = \mathbf{u}_t^\top L(\boldsymbol{\xi}^t)$$



$L(\boldsymbol{\xi})$ Piecewise linear



$L(\boldsymbol{\xi})$ Piecewise constant

[7] Zhang, Georghiou, Lygeros (under review); Bertsimas, Georghiou (under review);

Decision Rule Approximation

$$\begin{aligned}
&\text{minimize} \quad && \mathbb{E}\left(\sum_{t=1}^{T} \ell_t(\boldsymbol{x}_t, \boldsymbol{u}_t)\right) \\
&\text{subject to} \quad && \left.\begin{aligned}
&\boldsymbol{u}_t = \mathbf{U}_t^\top L(\boldsymbol{\xi}^t), \\
&\boldsymbol{x}_t \in \mathcal{X}_t,\ \boldsymbol{u}_t \in \mathcal{U}_t, \\
&\boldsymbol{x}_{t+1} = f_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t),
\end{aligned}\right\} \forall \boldsymbol{\xi} \in \Xi,\ t \in \mathcal{T}.
\end{aligned}$$

- Finite number of decisions $\mathbf{U}_t$
- Semi-infinite structure: robust optimization
- Resulting program: ***polynomial size in the input data***

Building Energy Management[8]

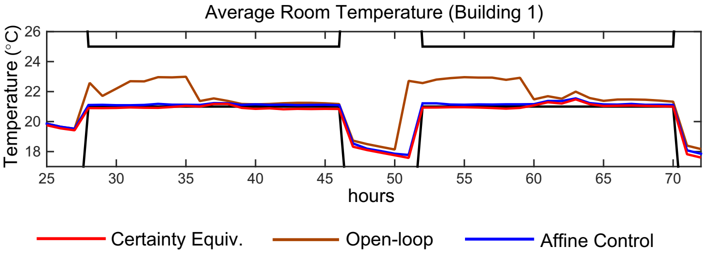
- 5 heterogenous buildings, 5 rooms each
- Sharing energy conversion units, storage units

Method	Winter (per room) $(\hat{\mu}, \hat{\sigma})$ Cost (CHF)	Violation (Kh)
Certainty Equiv.	$(34.0, 2.2)$	$(30.4, 1.5)$
Open-Loop	$(48.7, 1.7)$	$(5.6, 0.5)$
Affine Control	$(44.2, 1.1)$	$(5.5, 0.4)$

Method	Summer (per room) $(\hat{\mu}, \hat{\sigma})$ Cost (CHF)	Violation (Kh)
Certainty Equiv.	$(1.5, 0.3)$	$(7.4, 2.0)$
Open-Loop	$(5.2, 0.4)$	$(2.7, 0.6)$
Affine Control	$(4.7, 0.5)$	$(1.5, 0.4)$

[8] Darivianakis, Georghiou, Smith and Lygeros (under review)

Building Energy Management



Decision Rules: Cost / Benefit

Benefits of $u(\boldsymbol{\xi}) = \mathbf{u}^\top L(\boldsymbol{\xi})$

- Easy to construct
- Polynomial increase in problem size
- Allows for distributed solution methods (e.g., ADMM)

Drawbacks of $u(\boldsymbol{\xi}) = \mathbf{u}^\top L(\boldsymbol{\xi})$

- Unsuitable for highly non-linear dynamics $\boldsymbol{x}_{t+1} = f_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t)$
- Potentially restrictive in control designs

Dynamic Programming

Express problem as a **finite horizon dynamic program**

$$V_t(\boldsymbol{x}_t) = \min_{\boldsymbol{u}_t \in \mathcal{U}_t} \ell(\boldsymbol{x}_t, \boldsymbol{u}_t) + \mathbb{E}\left(V_{t+1}\big(f_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t)\big)\right), \quad \forall \boldsymbol{x}_t \in \mathcal{X}_t$$

Optimal policy

$$\boldsymbol{u}_t^*(\boldsymbol{x}_t) = \arg\min_{\boldsymbol{u}_t \in \mathcal{U}_t} \ell(\boldsymbol{x}_t, \boldsymbol{u}_t) + \mathbb{E}\left(V_{t+1}\big(f_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t)\big)\right), \quad \forall \boldsymbol{x}_t \in \mathcal{X}_t$$

Determine cost-to-go functions $V_t(\cdot)$

Dynamic Programming

Linear programming formulation

$$\begin{aligned}
\text{max} \quad & \int_{\mathcal{X}_1} c(\boldsymbol{x}_1) V_1(\boldsymbol{x}_1)\, \mathrm{d}\boldsymbol{x}_1 \\
\text{s.t.} \quad & V_t(\cdot) \in \text{Real valued functions}, \\
& V_t(\boldsymbol{x}_t) \le \ell_t(\boldsymbol{x}_t, \boldsymbol{u}_t) + \mathbb{E}\big(V_{t+1}(f_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t))\big), \\
& \forall \boldsymbol{x}_t \in \mathcal{X}_t,\ \boldsymbol{u}_t \in \mathcal{U}_t,\ \text{for } t = 1, \dots, T
\end{aligned}$$

Computing the optimal cost-to-go is computationally intractable

Approximate Dynamic Programming

$$\begin{aligned}
\max \quad & \int_{\mathcal{X}_1} c(\boldsymbol{x}_1)\widehat{V}_1(\boldsymbol{x}_1)\,\mathrm{d}\boldsymbol{x}_1 \\
\text{s.t.} \quad & \widehat{V}_t(\cdot) \in \text{Fixed degree polynomial}, \\
& \widehat{V}_t(\boldsymbol{x}_t) \leq \ell_t(\boldsymbol{x}_t, \boldsymbol{u}_t) + \mathbb{E}\big(\widehat{V}_{t+1}(f_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t))\big), \\
& \forall \boldsymbol{x}_t \in \mathcal{X}_t,\ \boldsymbol{u}_t \in \mathcal{U}_t,\ \text{for } t = 1, \ldots, T
\end{aligned}$$

Approximate policy

$$\widehat{\boldsymbol{u}}_t(\boldsymbol{x}_t) = \arg\min_{\boldsymbol{u}_t \in \mathcal{U}_t} \ell_t(\boldsymbol{x}_t, \boldsymbol{u}_t) + \mathbb{E}\big(\widehat{V}_{t+1}(f_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t))\big), \quad \forall \boldsymbol{x}_t \in \mathcal{X}_t$$

Distributed Control

Achieve distributed control structure

- Objective function has separable structure
- Input constraints has separable structure

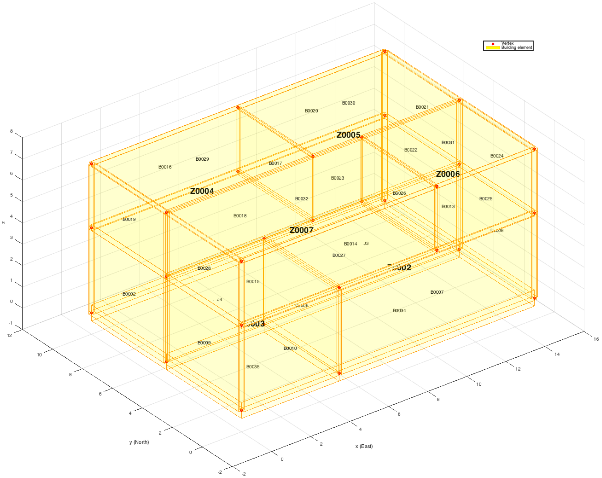
$$\widehat{\boldsymbol{u}}_t(\boldsymbol{x}_t) = \arg\min \; \ell_t(\boldsymbol{x}_t, \boldsymbol{u}_t) + \mathbb{E}\Big(\widehat{V}_{t+1}(f_t(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t))\Big)$$

$$\text{s.t. } \boldsymbol{u}_t = (u_{t,1}, \ldots, u_{t,n}) \in \mathcal{U}_t$$

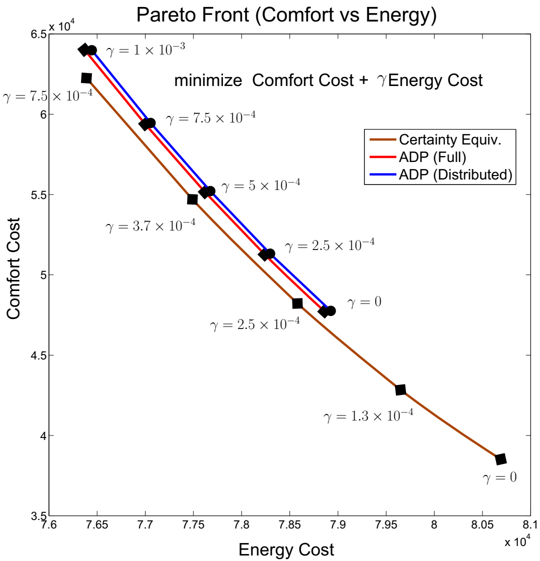
- Train $\widehat{V}_{t+1}(\cdot)$ for separable objective function structure
- Employ distributed solution methods (e.g., ADMM)

Numerical Results

- 7-room, 2-storey building



Numerical Results



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